SOME RESULTS OF DENSITY DETERMINATION IN GAS STREAMS ON THE BASIS OF DISPLACEMENT INTERFEROGRAMS

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Results are shown of a quantitative evaluation of displacement interferograms taken in supersonic streams. A formula is derived for calculating the error which refraction of light introduces into a density measurement. A comparison is made between experiment and theory.

Large gaseous inhomogeneities are nowadays examined with the aid of a two-beam mirror-type displacement interferometer operating in convergent light beams [1]. This instrument is portable, simple in construction as well as setup, and shakeproof; it also yields white interferograms, which are a necessary accessory in any study of gas streams with density jumps, inasmuch as jumps of the order of interference fringes during transition through density jumps ~~cannot always be~~ calculated theoretically.

The path of the light rays in this instrument is as follows. A light source behind a slit is placed at the center of curvature of a spherical mirror whose dimensions determine the dimensions of the interference field. The divergent light falls on the spherical mirror surface and, after reflection, proceeds into the optical system of the interferometer, where it transverses the inhomogeneity field twice.

In many cases one may discount the slight convergence of the light beam and regard it as parallel.

The rays of a convergent beam lie in vertical planes which intersect a horizontal and axially symmetric gas stream along ellipses. The compression ratio of these ellipses is defined as follows:

$$\delta = [(1 - a^2 r_d^2 / L^2 l^2)/(1 + a^2/L^2)]^{1/2} . \tag{1}$$

In practice L = 4-17 m.

Formula (1) has been derived from the equation of an ellipse in polar coordinates, with the pole at an arbitrary point on the major axis. The ellipses lie in layers bounded by planes x = c and

$$\Delta \approx 2 r_d a / L \tag{2}$$

thick.

In this experiment, which involved a supersonic stream with the Mach number $M_\infty = 2$ around a cylinder with a hemispherical nose, the layer thickness was $\Delta = 2 \cdot 10^{-4}$ m at section x = 1.25 R. Moreover, the plane intersecting this stream along a circle passed near the nose of the model. The compression ratio δ in this section was such as to make the difference between the semiaxes of the ellipse equal to $6 \cdot 10^{-6}$ m. At a distance $2 \cdot 10^{-4}$ m the density along the x axis was almost constant and the ellipses at a sufficiently far distance a became almost circles.

On account of this, the interferograms could be evaluated by the method applicable to a parallel beam.

Correction for the Refraction of Light in the Analysis of an Axially Symmetric Inhomogeneity. In our case the refractive index is $n = n(x,\ r = \sqrt{y^2 + z^2})$. A light ray falling on an inhomogeneity normally to the x axis intersects that inhomogeneity along a three-dimensional contour with the curvature

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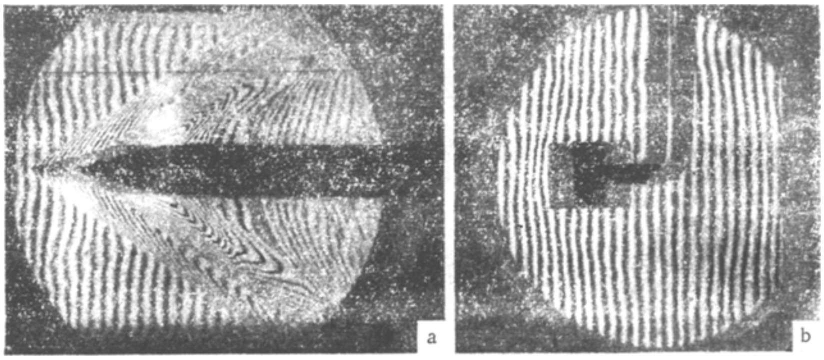

Fig. 1. Displacement interferogram of a supersonic stream (a) and of a hypersonic stream (b) around a cylinder, with a full displacement of the wave front.

$$\varkappa = \frac{1}{n}\left[\left(\frac{\partial n}{\partial r}\right)^2 \sin^2 i + \left(\frac{\partial n}{\partial x}\right)^2\right]^{1/2} \tag{3}$$

and the twist

$$\tau = -\frac{1}{2rn^2\varkappa^2}\frac{\partial n}{\partial r}\frac{\partial n}{\partial x}\sin 2i. \tag{4}$$

Equations (3) and (4) have been derived from the conventional expressions for $\varkappa$ and τ [2].

Only the effect of component $\partial n/\partial r$ on the correction for refraction needs to be considered in an analysis of the radial density profile. Letting $\partial n/\partial x = 0$, we have

$$\varkappa = \frac{1}{n}\frac{\partial n}{\partial r}\sin i, \quad \tau = 0,$$

where $\varkappa$ and $\partial n/\partial r$ represent the magnitudes of vectors $\vec{\varkappa}$ and $\partial \mathbf{n}/\partial \mathbf{r}$, respectively. The sign of the curvature is determined by the sign of vector $\partial \mathbf{n}/\partial \mathbf{r}$.

An elementary angular displacement of a ray is $d\varepsilon/\varkappa ds$.

Since in the given case $ds = dr/\cos i$, hence

$$d\varepsilon = \frac{1}{n}\frac{\partial n}{\partial r}\,\mathrm{tg}\, i\, dr. \tag{5}$$

For a ray parallel to the z axis

$$\mathrm{tg}\, i \approx y/\sqrt{r^2 - y^2}. \tag{6}$$

Equality (6) is exact only with an invariant $nr(\sin i) = n_0 r_0 \sin i_0$ along the ray, where subscript 0 refers to the point at which the ray enters an inhomogeneity. Then Eq. (5) will yield the integral of stellar refraction

$$\varepsilon = \int_n^{n_0} \frac{1}{n}\frac{\sin i\, dn}{\sqrt{1 - (n_0 r_0/nr)^2 \sin^2 i_0}} \text{ [3]}.$$

From Eqs. (5) and (6) follows

$$\varepsilon \approx 2\int_y^{r_0} \frac{1}{n}\frac{\partial n}{\partial r}\frac{y\,dr}{\sqrt{r^2 - y^2}}. \tag{7}$$

Integral (7) is used for quantitative optical studies of gas dynamics [4]. It is assumed there that the ray deflections before and behind the xoy plane are equal. The described method makes it feasible to determine the angle of refraction for any incident angle at an inhomogeneity.

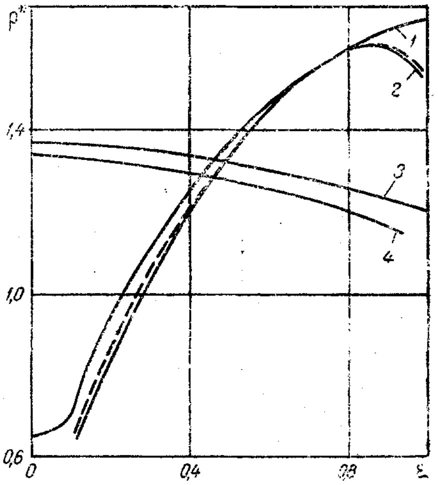

Fig. 2

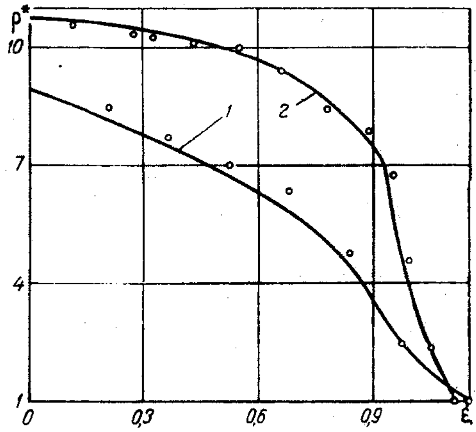

Fig. 3

Fig. 2. Density lines in streams around a cone and a spherically blunted cylinder.

Fig. 3. Density lines in a stream shown in Fig. 1b.

When the instrument is focused on the center of an axially symmetric inhomogeneity, then the error in the density determination within some annular zone s, due to the refraction of light, is

$$\Delta\rho_s = \frac{\partial\rho}{\partial r}\Big|_s b. \tag{8}$$

Angle ε is subtended by two tangents to the ray, at the entrance to and the exit from the inhomogeneity, which intersect at the center of that inhomogeneity. By virtue of the optical geometry of a displacement interferometer, we have

$$b \approx \varepsilon\left(2l + \frac{1}{2}\sqrt{r_0^2 - y^2}\right). \tag{9}$$

Formula (9) is based on the assumption that $y/L \ll 1$.

We will now apply the well-known method of annular zones [4]. In that case Eqs. (7), (8), and (9) with $\Delta n = k\Delta\rho$ yield

$$\Delta\rho_s^* \approx k\rho_\infty \left(4l + \sqrt{r_0^2 - r_s^2}\right) r_s \frac{\partial\rho^*}{\partial r}\Big|_s \sum_{i=1}^{s} \left(\frac{\partial\rho^*}{\partial r}\right)_i \ln\frac{r_{i-1} + \sqrt{r_{i-1}^2 - r_s^2}}{r_i + \sqrt{r_i^2 - r_s^2}} \tag{10}$$

Formula (10) applies to the case where the light beam traverses the analyzed gas stream twice.

Test Results. Quantitative measurements with a mirror-type displacement interferometer are best made with a full displacement of the wave front. In this case the perturbed wave front interferes with the unperturbed one, and ordinary interference takes place as in the Zehnder–Mach instrument. If the magnitudes of the density gradient around the test model are large, however, then one prefers to make the displacement small. Otherwise the visibility of the interference fringes would be greatly reduced. In a stream at $M_\infty = 2$ and $\rho_\infty = 0.476$ kg/m^3 around a spherically blunted cylinder, a displacement of the wave front by an amount equal to 0.3 R will appreciably reduce the visibility of fringes around the density jump and will make it impossible to determine the density in all parts of the interferogram.

A displacement interferogram of the flow around a narrow cone with an angle $\beta_c = 15°$ under the said conditions is shown in Fig. 1a (a full displacement is possible here), a displacement interferogram for the flow at $M_\infty = 11.16$ and $\rho_\infty = 2.62 \cdot 10^{-3}$ kg/m^3 around a cylinder with a flat endface is shown in Fig. 1b. In this case, too, a full displacement of the wave front is possible.

The results of a quantitative evaluation of this displacement interferogram are shown in Fig. 2. The method of evaluation has been described in [5]. The ordinates represent the dimensionless density ρ^*, the abscissas represent the dimensionless space coordinate ξ. Curve 1 represents the theoretical values at section $x = 1.25R$ in the stream at $M_\infty = 2$ [6] around the cylinder with a hemispherical nose, curve 2 is based on a displacement interferogram for the same stream but with the interferometer set up for horizontal fringes and a small vertical displacement of the wave front. Such an interferogram has been shown in [5]. The dashed line represents the density profile calculated from the displacement interferogram with a correction for refraction. This correction has been determined according to formula (10). The maximum deviation of the test curve from the theoretical curve is 8% near the shock wave and 9.5% near the boundary layer with a correction for refraction or 13% without such a correction. Curve 3 is the theoretical density profile at $M_\infty = 2$ near the narrow cone with $\beta_c = 15°$ according to [7], curve 4 is based on the interferograms with a full displacement of the wave front (Fig. 1) and a small vertical displacement. It is to be noted that the wave front at section $x = c$ on the interferogram in Fig. 1 coincides completely with the wave front reproduced from the interferogram with a small vertical displacement. The maximum difference between the test curve and the theoretical curve is 6%.

The displacement interogram in Fig. 1b has been evaluated quantitatively and the results are shown in Fig. 3. Curve 1 represents the density profile at section $x = 0.006R$ and curve 2 represents the density profile at section $x = -0.2R$. The x axis runs in the direction of flow, with the origin on the cylinder endface. The solid lines have been calculated by the method of annular zones, with the density in each thin annulus assumed constant. The points indicate the values of density calculated by the V. A. Emel'yanov method, i.e., by approximating the measured curve through each annulus with a Lagrange polynomial [8]. We note that no sudden density jump occurs in the shock wave in such a stream. No shock wave is seen in Fig. 1b, because the density ρ_∞ is very low. The sensitivity of a displacement interferometer can be raised with respect to the visualization of a density jump, however, if the wave front is displaced somewhat either horizontally or vertically when the instrument is set up for an infinitely wide fringe. A shock wave becomes, indeed, clearly visible in such a setup.

A luminous region is visible during shadowgraphing such a stream around the cylinder endface. Spectral measurements have shown, however, that nitrogen, oxygen, and hydrogen remain unexcited in this region. Luminosity is caused by the combustion products in the preheating electric arc. For this reason, curves 1 and 2 in Fig. 3 have been calculated according to the conventional relation between the refractive index and the density of air. The correction for refraction amounts to 1% around a shock wave and to a fraction of a percent behind a shock wave.

NOTATION

L	is the distance from the center of curvature of the spherical interferometer mirror to the symmetry axis x of the analyzed model;
r_d	is the radius of an axially symmetric inhomogeneity with the origin at point d;
t	is the distance from point d to the intersection point between the tangents to the stream at the endpoints of radii r_d;
a	is the distance from the intersection point between the x axis and the major axis of the ellipse with a compression ratio $\delta = 1$ to such an intersection point with the major axis of an ellipse whose compression ratio is $\delta < 1$;
i	is the angle between vector $\partial\mathbf{n}/\partial\mathbf{r}$ and the direction of a light ray;
n	is the refractive index;
ds	is a differential of arc s along which light travels;
b	is the linear deflection of a light ray at the center of an inhomogeneity along the y axis;
l	is the distance from the spherical mirror along the x axis;
k	is the Gladstone–Dale constant;
$\rho^* = \rho/\rho_\infty$;	
ρ	is the density at some perturbation point;
ρ_∞	is the density of the oncoming stream;
$\xi = (r - r_b)/(r_0 - r_b)$;	
r	is the local radius;
r_b	is the radius of the axially symmetric model surface;
r_0	is the radius of a shock wave in a stream with an extinct shock wave, or the radius of a perturbation near the model in a stream with a persistent shock wave;
β_c	is half the cone angle;

R is the radius of the cylinder;

r_i is the radius of annulus i;

M_∞ is the Mach number of the oncoming stream.

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